



Making an Asymmetric PAKE Quantum-Annoying by Hiding Group Elements

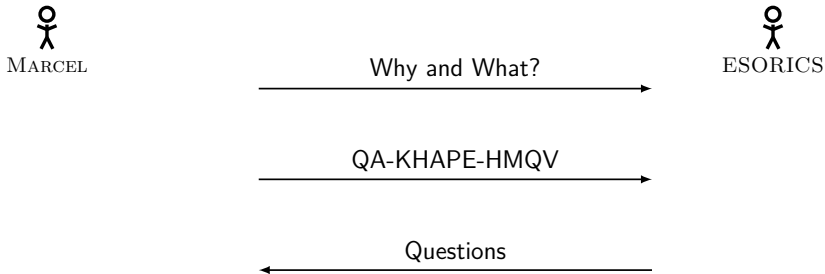
ESORICS, The Hague, 2023

Marcel Tiepelt, Edward Eaton, Douglas Stebila

Making an Asymmetric PAKE
Quantum-Annoying by Hiding Group Elements

Making an Asymmetric PAKE

Quantum-Annoying by Hiding Group Elements

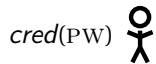


Typical Client-Server Authentication



Client

Registration




Server

Typical Client-Server Authentication

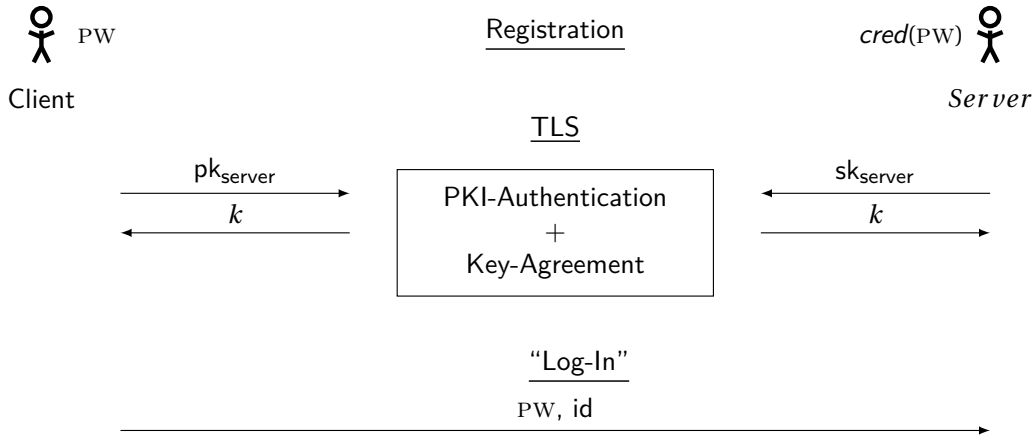
 PW
Client

Registration

cred(PW) 
Server


Attacker


Typical Client-Server Authentication



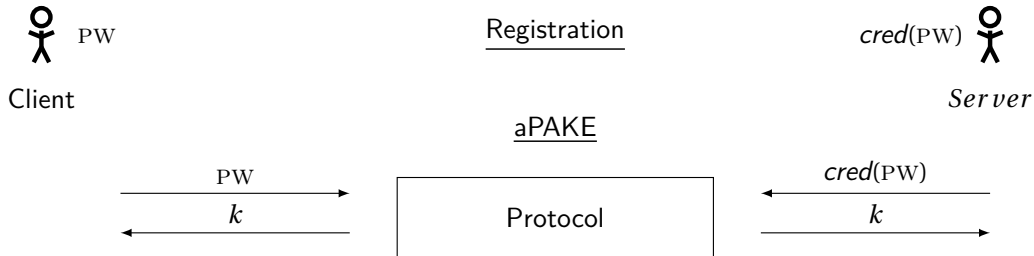
Asymmetric Password Authenticated Key Exchange

 PW
Client

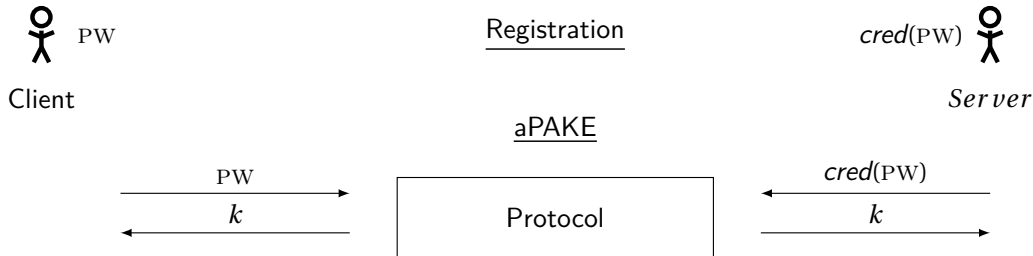
Registration

cred(PW) 
Server

Asymmetric Password Authenticated Key Exchange

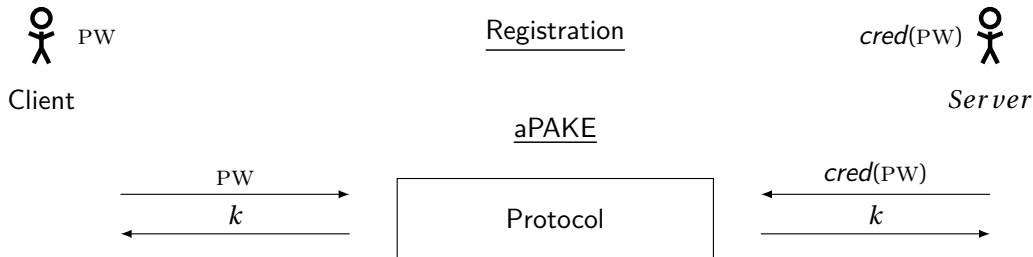


Asymmetric Password Authenticated Key Exchange



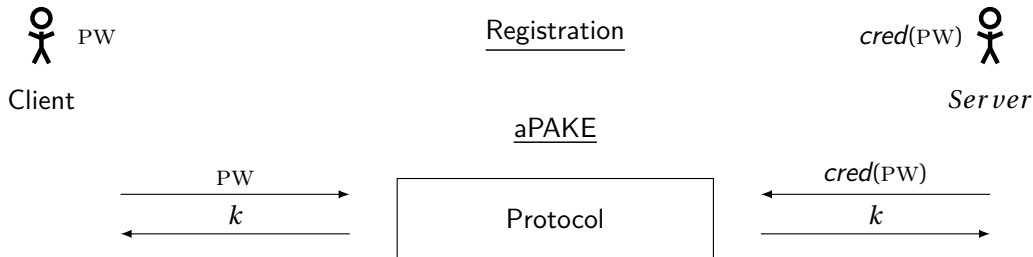
$$\text{Adv} \leq \frac{\# \text{Online Interactions}}{\text{PW-Space}} + \text{Intractability Assumption}$$

Asymmetric Password Authenticated Key Exchange



$$Adv \leq \frac{\#Online\ Interactions}{PW\text{-Space}} + DLOG$$

Asymmetric Password Authenticated Key Exchange



$$Adv \leq \frac{\#Online\ Interactions}{PW\text{-Space}} + \mathbf{DLOG}$$

Bad News: *Quantum computers
might break DLOG.*

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Good News: *Quantum computing
appears to be expensive!*

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*Force adversary to use a lot of
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Bad News: *Quantum computers
might break DLOG.*

Good News: *Quantum computing
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*Force adversary to use a lot of
quantum computing!*

$1 \times \text{DLOG}$ total



$1 \times \text{DLOG}$
per password guess

Quantum Annoying'ness ²

Security

$$\text{Adv} \leq \frac{\text{\#Online Interactions}}{\text{PW-Space}} + \frac{\text{\#DLOG's}}{\text{PW-Space}}$$

Model

- DLOG Oracle
- GGM
- BPR¹

Limitations

- Only DLOG oracle
- Multiple DLOG's harder than one DLOG

²Eaton and Stebila 2021, "The "Quantum Annoying" Property of Password-Authenticated Key Exchange Protocols"

¹Bellare, Pointcheval, and Rogaway 2000, "Authenticated Key Exchange Secure against Dictionary Attacks"



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aPAKE and Quantum-Annoying'ness



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QA-KHAPE-HMQV

Questions

KHAPE-HMQV³ — simplified



Client

Registration

$(a, A), (b, B)$ fresh AKE keys

$e \leftarrow IC_1.E(PW, a, B, \quad)$

store $cred(PW) = (b, A, e \quad)$



Server



³Gu, Jarecki, and Krawczyk 2021, “KHAPE: Asymmetric PAKE from Key-Hiding Key Exchange”

KHAPE-HMQV³ — simplified



Client

Registration

$(a, A), (b, B)$ fresh AKE keys

$e \leftarrow IC_1.E(PW, a, B, \quad)$

store $cred(PW) = (b, A, e \quad)$



Server

On input: PW

aPAKE

$\leftarrow Y, e$

On input: $cred(PW)$

$y \stackrel{\$}{\leftarrow} \mathbb{Z}_p, Y \leftarrow g^y$

³Gu, Jarecki, and Krawczyk 2021, “KHAPE: Asymmetric PAKE from Key-Hiding Key Exchange”

KHAPE-HMQV³ — simplified



Client

Registration

$(a, A), (b, B)$ fresh AKE keys

$e \leftarrow IC_1.E(PW, a, B, \quad)$

store $cred(PW) = (b, A, e \quad)$



Server

On input: PW

$a, B \quad \leftarrow IC_1.D(PW, e)$

$x \xleftarrow{\$} \mathbb{Z}_p, X \leftarrow g^x$

$\sigma \leftarrow DH(a, B, x, Y)$

$\tau \leftarrow F(*, \sigma)$

aPAKE

$\longleftarrow Y, e$

$\longrightarrow X, \tau$

On input: $cred(PW)$

$y \xleftarrow{\$} \mathbb{Z}_p, Y \leftarrow g^y$

³Gu, Jarecki, and Krawczyk 2021, "KHAPE: Asymmetric PAKE from Key-Hiding Key Exchange"

KHAPE-HMQV³ — simplified



Client

Registration

$(a, A), (b, B)$ fresh AKE keys

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store $cred(PW) = (b, A, e \quad)$



Server

On input: PW

$a, B \leftarrow IC_1.D(PW, e)$

$x \xleftarrow{\$} \mathbb{Z}_p, X \leftarrow g^x$

$\sigma \leftarrow DH(a, B, x, Y)$

$\tau \leftarrow F(*, \sigma)$

Check γ

aPAKE

$\leftarrow Y, e$

$\rightarrow X, \tau$

$\leftarrow \gamma$

On input: $cred(PW)$

$y \xleftarrow{\$} \mathbb{Z}_p, Y \leftarrow g^y$

$\sigma' \leftarrow DH(b, A, X, y)$

Check τ


$\gamma \leftarrow F(*, \sigma')$

³Gu, Jarecki, and Krawczyk 2021, "KHAPE: Asymmetric PAKE from Key-Hiding Key Exchange"


KHAPE-HMQV³ — simplified



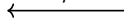
Y, e



X, τ



γ



³Gu, Jarecki, and Krawczyk 2021, “KHAPE: Asymmetric PAKE from Key-Hiding Key Exchange”

KHAPE-HMQV³ — simplified




Not Quantum Annoying


Attacker:

- query $DLOG(X) \rightarrow x$,
 - check PW_i
- $\rightsquigarrow IC.D(PW_i, e) \rightarrow a_i, B_i$
until $\tau = F(DH(a_i, B_i, x, Y))$

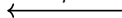
Y, e



X, τ



γ



³Gu, Jarecki, and Krawczyk 2021, “KHAPE: Asymmetric PAKE from Key-Hiding Key Exchange”

QA-KHAPE-HMQV – simplified



Client

Registration

$(a, A), (b, B)$ fresh AKE keys

$e \leftarrow IC_1.E(PW, a, B, \quad)$;



Server

On input: PW

$a, B \leftarrow IC_1.D(PW, e)$

$x \stackrel{\$}{\leftarrow} \mathbb{Z}_p, X \leftarrow g^x$

$\sigma \leftarrow DH(a, B, x, Y)$

$\tau \leftarrow F(*, \sigma_{Client})$

Check γ

aPAKE

$\leftarrow Y, e$

$\xrightarrow{X, \tau}$

$\leftarrow \gamma$

On input: $cred(PW)$

$y \stackrel{\$}{\leftarrow} \mathbb{Z}_p, Y \leftarrow g^y$

$\sigma' \leftarrow DH(b, A, X, y)$

Check τ

$\gamma \leftarrow F(*, \sigma')$

QA-KHAPE-HMQV – simplified



Client

Registration

$(a, A), (b, B)$ fresh AKE keys
 $e \leftarrow \text{IC}_1.E(\text{PW}, a, B, \text{sk}); \text{sk} \leftarrow \{0, 1\}^k$



Server

On input: PW

$a, B, \text{sk} \leftarrow \text{IC}_1.D(\text{PW}, e)$
 $x \leftarrow \mathbb{Z}_p, X \leftarrow g^x$
 $\sigma \leftarrow \text{DH}(a, B, x, Y)$
 $c_X \leftarrow \text{IC}_2.E(\text{sk}, X)$
 $\tau \leftarrow F(*, \sigma_{\text{Client}})$

Check γ

aPAKE

$\leftarrow Y, e$

$\rightarrow c_X, \tau$

$\leftarrow \gamma$

On input: $\text{cred}(\text{PW})$

$y \leftarrow \mathbb{Z}_p, Y \leftarrow g^y$

$X \leftarrow \text{IC}_2.D(\text{sk}, c_X)$

$\sigma' \leftarrow \text{DH}(b, A, X, y)$

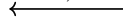
Check τ

$\gamma \leftarrow F(*, \sigma')$

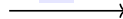
QA-KHAFE-HMQV – simplified



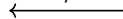
Y, e



c_X, τ



γ



QA-KHAPE-HMQV – simplified



Not Quantum Annoying

Attacker can

- query ~~DLOG(X)~~ x ,
 - check PW_i
- $\rightsquigarrow IC.D(PW_i, e) \rightarrow a_i, B_i, sk_i,$
 $IC.D(sk_i, c_X) \rightarrow X_i,$
query $DLOG(X_i) \rightarrow x_i$
until $\tau = F(DH(a_i, B_i, x_i, Y))$

Y, e

c_X, τ

γ

Takeaway

PAKEs are great

PAKE and Quantum-Annoying'ness

Single **DLOG**'s vulnerable

Quantum Annoyingness

QA-KHAPE-HMQV

Some quantum resistance,
if **many DLOG's** are **expensive**

Ideal Cipher

Quantum Annoying aPAKE “for free”



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← Questions →



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“Making an Asymmetric PAKE Quantum-Annoying by Hiding Group Elements”

~~[A Short Link to the Paper](#)~~

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