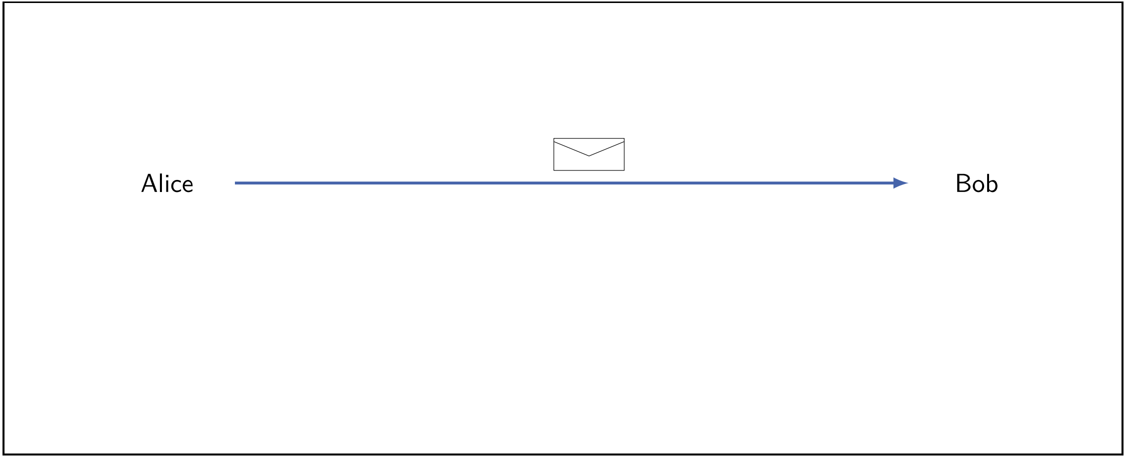
Costing Adversaries on Quantum-secure Cryptography

A Dissertation Talk

Marcel Tiepelt | January 23rd, 2025

Reviewers Jörn Müller-Quade
Douglas Stebila
Daniel Loebenberger

The Internet



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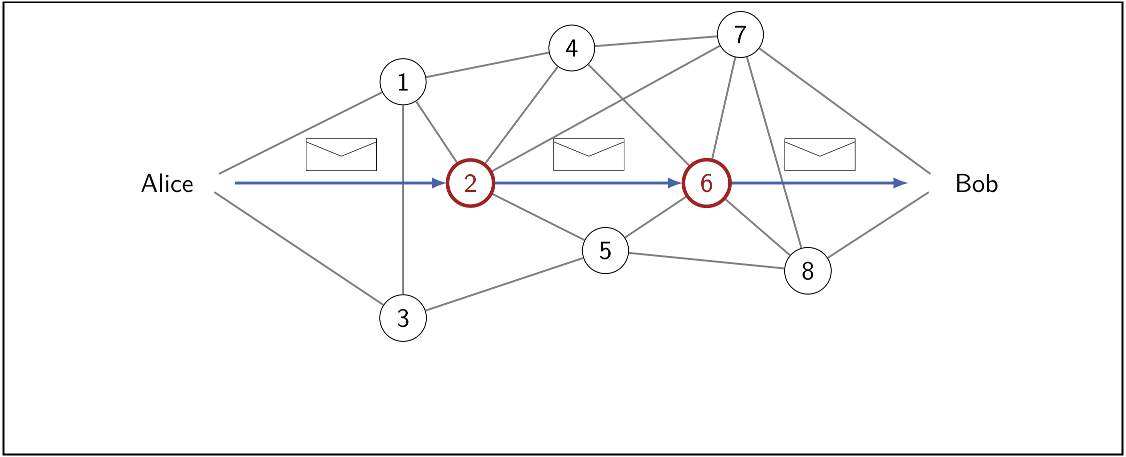

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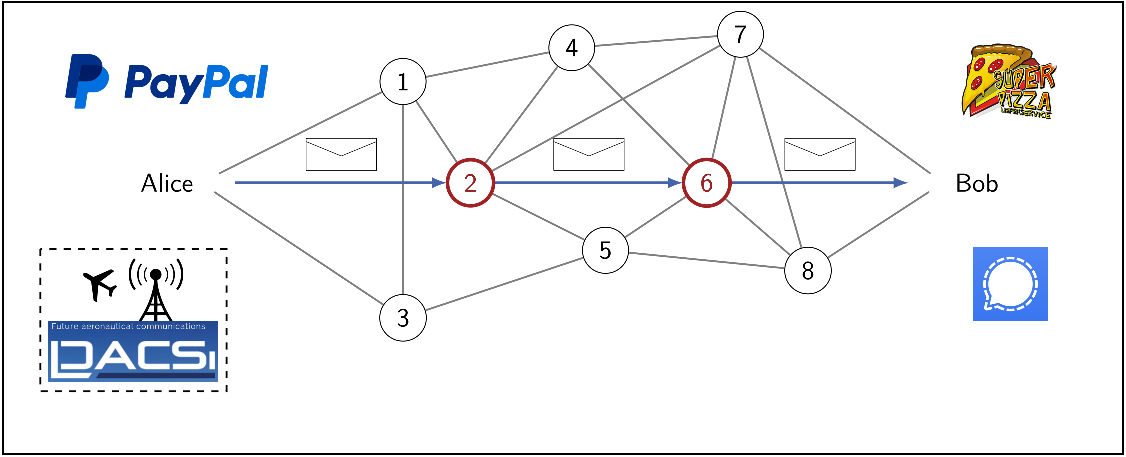


The Internet

Use of encryption

> 90% of internet traffic[1]

> 95% of websites on Google[2]

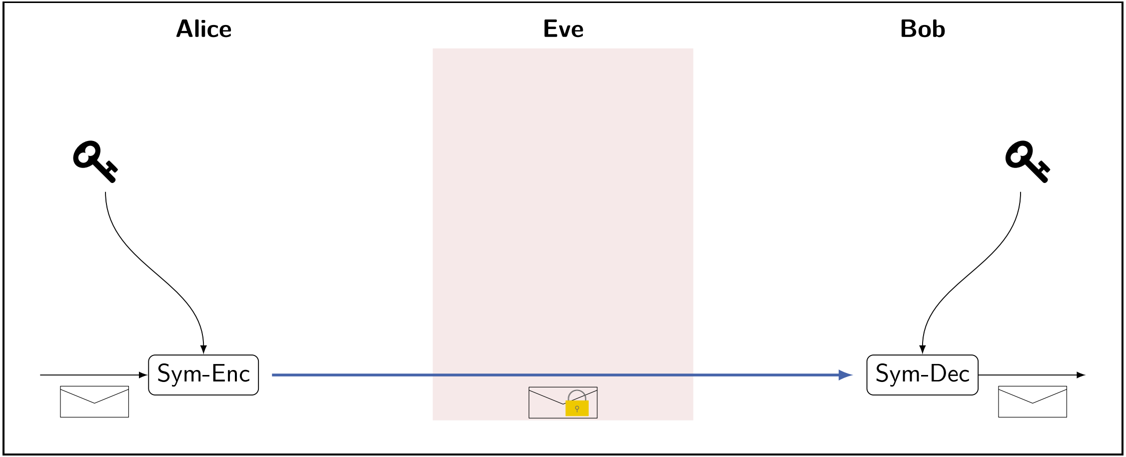
> 99% of browsing time on Google Chrome[3]

[1] Lee et al., TLS 1.3 in Practice:How TLS 1.3 Contributes to the Internet
[2,3] `https://serpwatch.io/blog/ssl-stats/`

The Internet — simplified

Alice **Eve** **Bob**

The Internet — simplified



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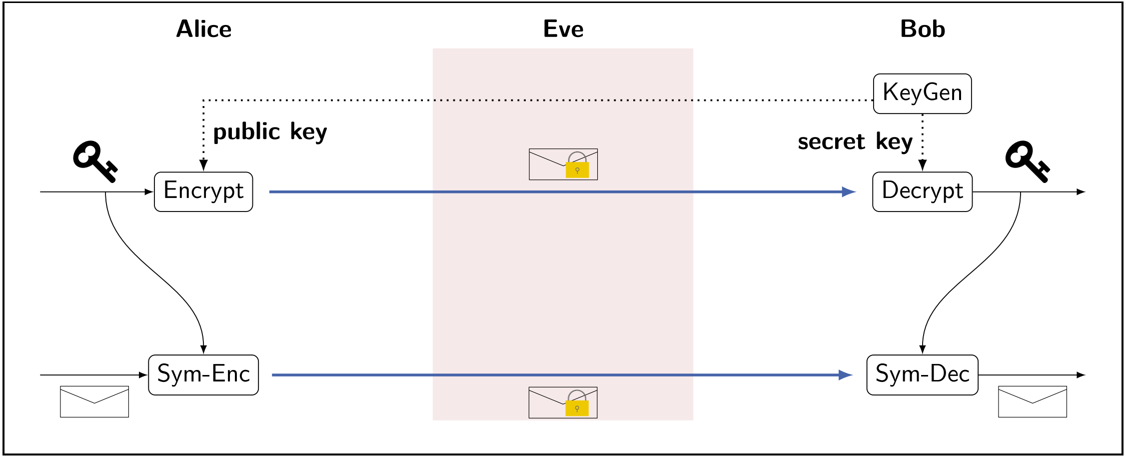

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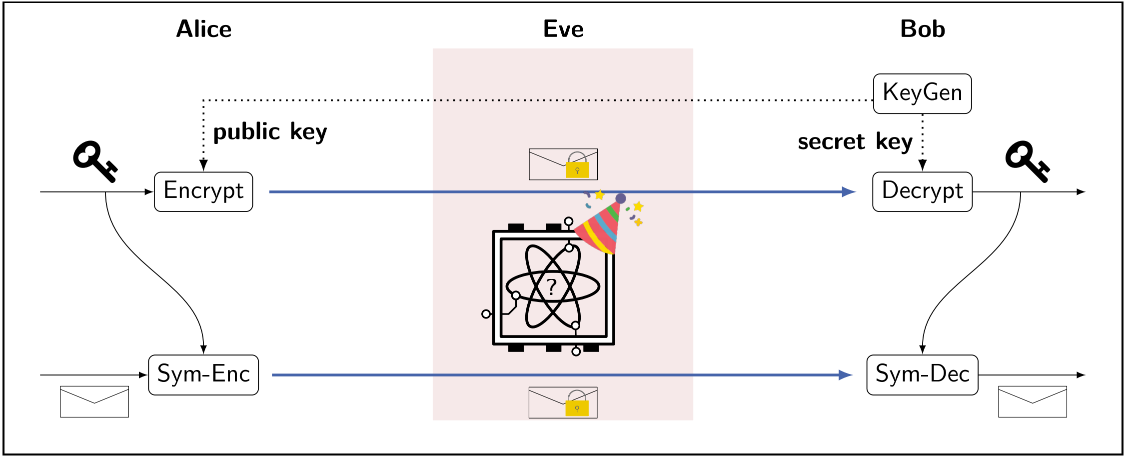

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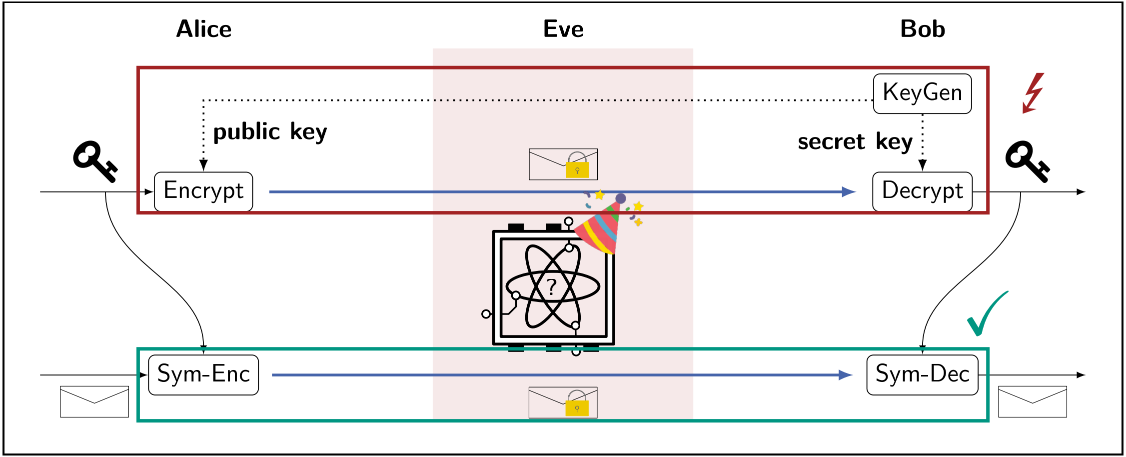

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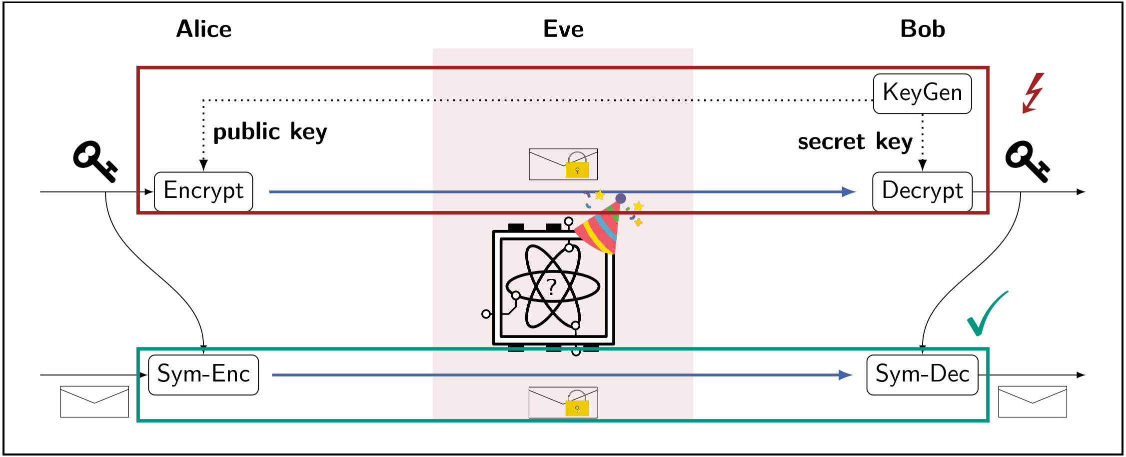


How does quantum computing affect the security of public-key cryptography?

Today's talk

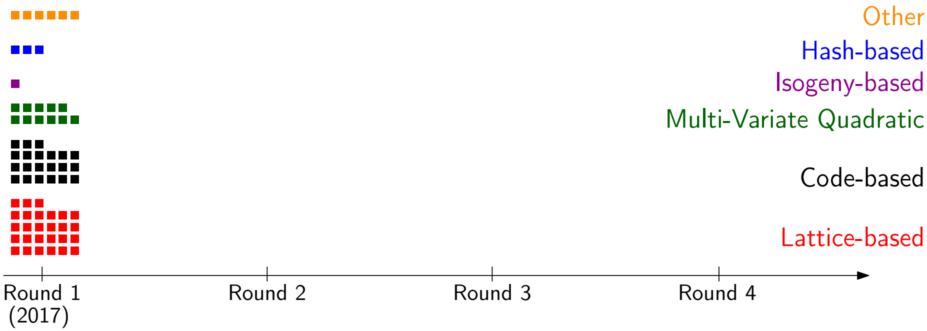
The Internet

I. Advancements in quantum-secure cryptography

II. When is a cryptographic protocol quantum-secure?

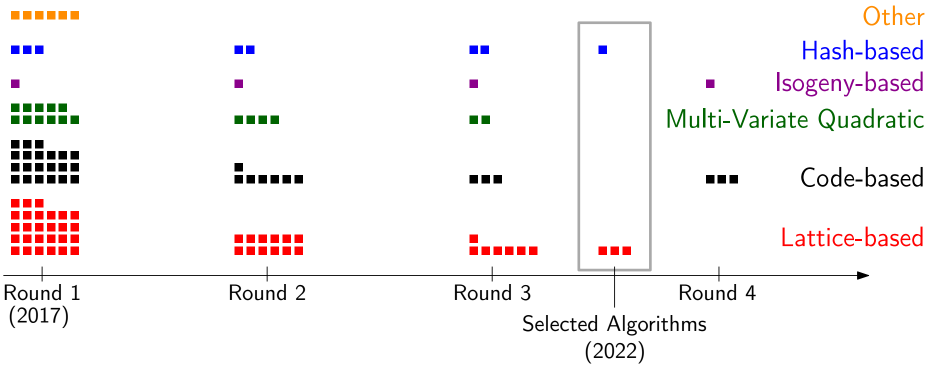
III. The impact of quantum lattice enumeration

NIST post-quantum standardization



Uncertainty in security

NIST post-quantum standardization



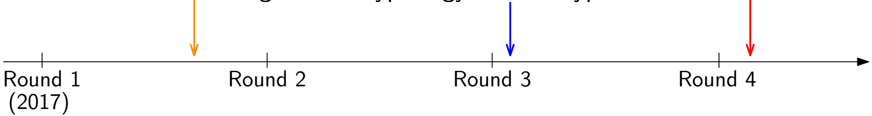
Uncertainty in security $\xrightarrow{\text{public evaluation}}$ *Confidence* in security

Contributions in the thesis: Part I

Exploiting Decryption Failures in Mersenne Number Cryptosystems
PKC at AsiaCCS 2020

Lattice Enumeration in Limited Depth
Advances in Cryptology – CRYPTO 2024

Forging SPHINCS$^+$-Haraka Signatures on a Fault-Tolerant Quantum Computer
Progress in Cryptology – Latincrypt 2021



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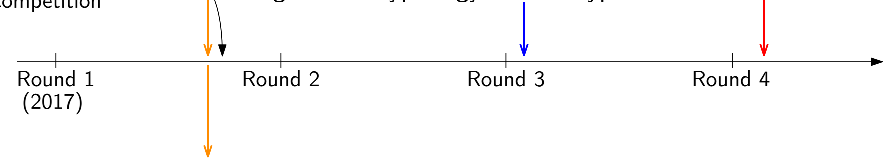
Mersenne-number cryptosystems dropped out of competition



Full security break from decryption failures.

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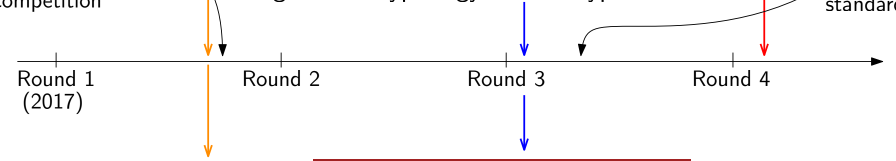

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Identify best point of attack and estimate concrete resources.

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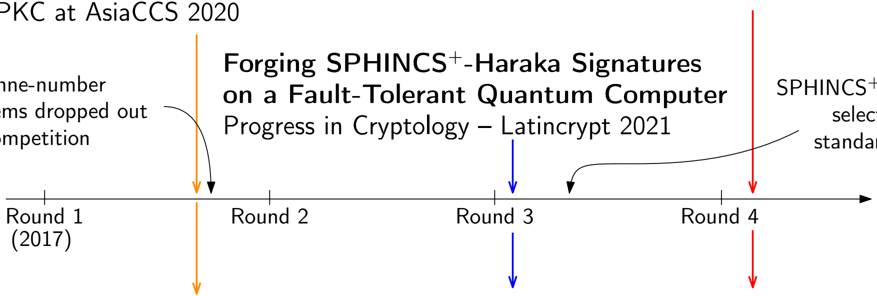
Mersenne-number cryptosystems dropped out of competition

SPHINCS^{+}-Haraka not selected for standardization



Full security break from decryption failures.

Identify best point of attack and estimate concrete resources.

Lattice-based schemes: Lower bound cost of “best” attack

Contributions in the thesis: Part II

Making an Asymmetric PAKE Quantum-Annoying by Hiding Group Elements
ESORICS 2023

Post-Quantum Ready Key Agreement for Aviation
Communications in Cryptology 2024

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Quantum-Annoying: Intermediate security for Password Authenticated Key Exchange

↑

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Contributions in the thesis: Part II

Quantum-Annoying: Intermediate security for Password Authenticated Key Exchange

Quantum-secure data-link for civil aviation from NIST post-quantum schemes.

Making an Asymmetric PAKE Quantum-Annoying by Hiding Group Elements
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Protocol under standardization by ICAO

Contributions highlighted in this talk

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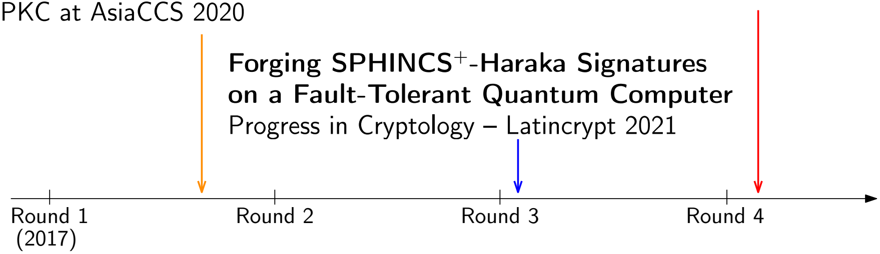

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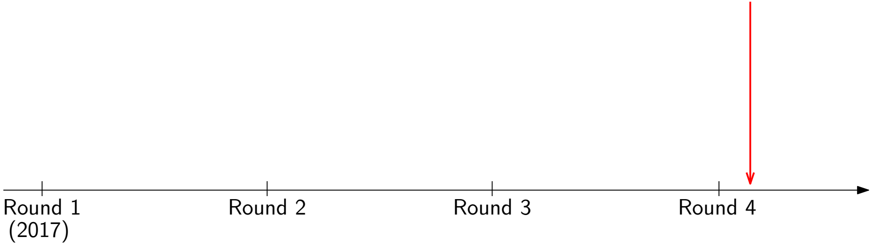

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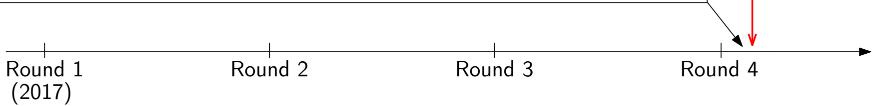
Lattice Enumeration in Limited Depth
Advances in Cryptology – CRYPTO 2024

August 2024, Federal Information Processing Standards:

- FIPS 203: Kyber
- FIPS 204: Dilithium



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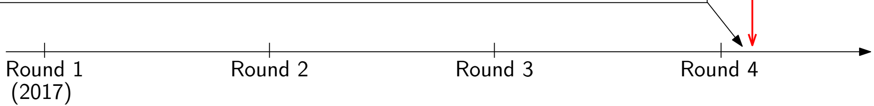
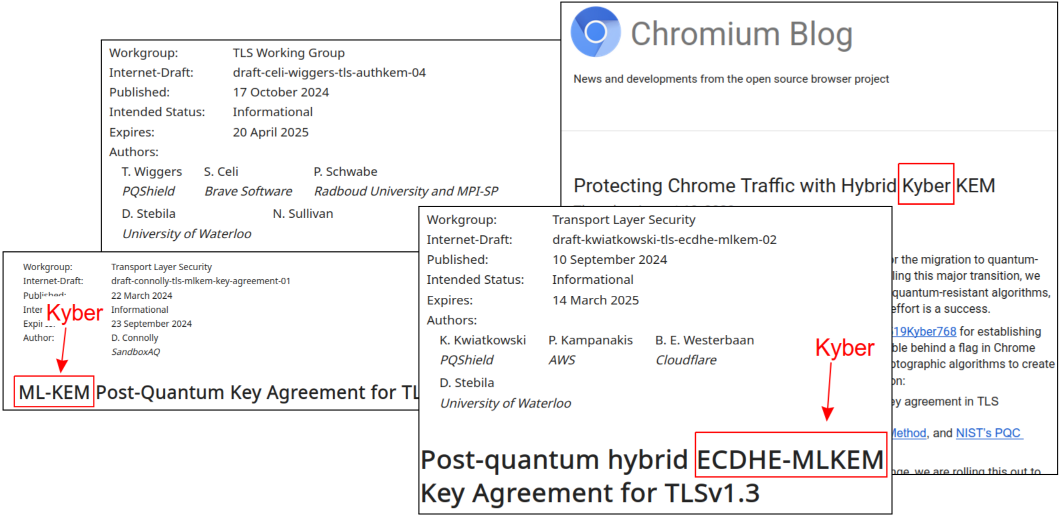


- Goal: Lower bound cost of “best” attack on lattice-based cryptography
- Analysis-Tool[a], Kyber as case study example

[a] available on Github

RFC = technical documentation and development of the internet



Today's talk

The Internet

I. Advancements in quantum-secure cryptography

II. When is a cryptographic protocol quantum-secure?

III. The impact of quantum lattice enumeration

When is a cryptosystem quantum-secure?

1) A cryptosystem is secure, if a certain computational problem is *difficult*.

When is a cryptosystem quantum-secure?

Kyber ↓ Shortest Vector Problem (SVP) ↓

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When is a cryptosystem quantum-secure?

Kyber → cryptosystem; Shortest Vector Problem (SVP) → computational problem

1) A cryptosystem is secure, if a certain computational problem is *difficult*.

2) Computational problem is believed to be *difficult*, if the best algorithm requires an infeasible amount of resources to solve it.

When is a cryptosystem quantum-secure?

Kyber

Shortest Vector Problem (SVP)

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Justification

What is the best algorithm to solve SVP?

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Lattice-reduction performs significantly better than other known algorithms.

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- Leading cost is **enumeration** or sieving[1]

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- Limitation: For quantum enumeration only asymptotic upper bound[2,3] known

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We don't know for sure.

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! Concrete cost of quantum enumeration not clear

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Why analyzing lattice enumeration matters

Lattice Enumeration in Limited Depth
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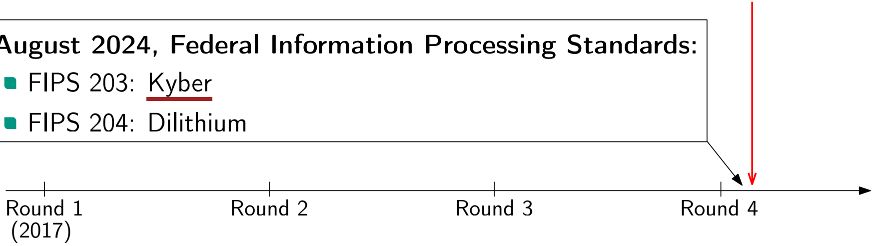
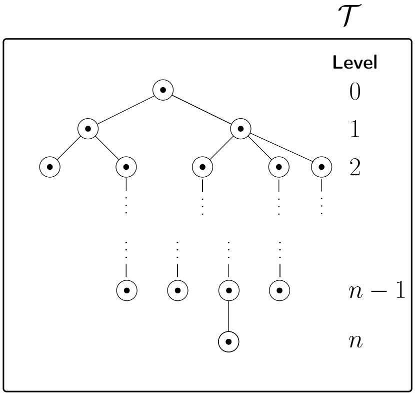


Concrete **security** of cryptographic standards remains **unknown**.

Lattice enumeration algorithm

Classical **enumeration** with extreme pruning[1]

- Search space is *n*-dimensional lattice



Lattice enumeration algorithm

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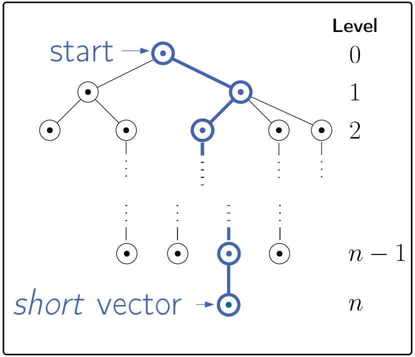
- **DFS** over enumeration tree

Depth First Search on $\mathcal{T}$



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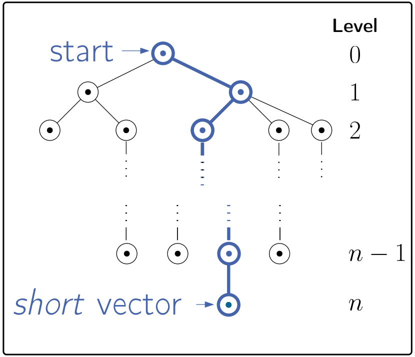

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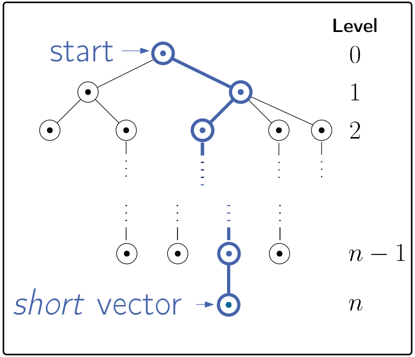
DFS as repetition of quantum walks[2]

$$\#\text{QW} \times \underbrace{O\left(\sqrt{\#\mathcal{T}} \cdot n\right) \times \overbrace{\mathcal{W}}^{\text{quantum operator}}}_{\text{quantum walk}}$$

Depth First Search on $\mathcal{T}$



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[2] Montanaro 2018, Quantum-Walk Speedup of Backtracking Algorithms

When is a cryptosystem quantum-secure?

Kyber

Shortest Vector Problem (SVP)

quantum
enumeration ✓

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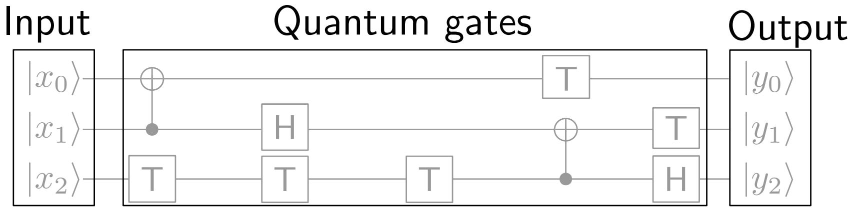
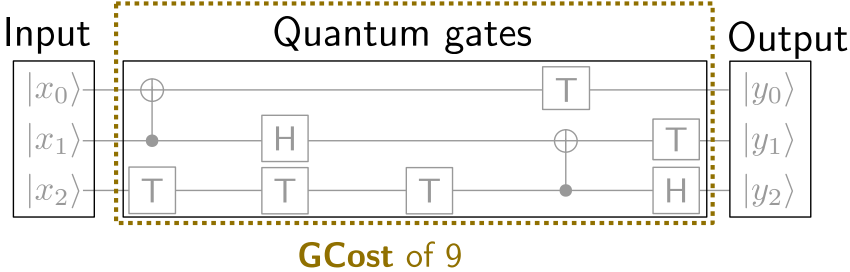
Resources: The quantum circuit model



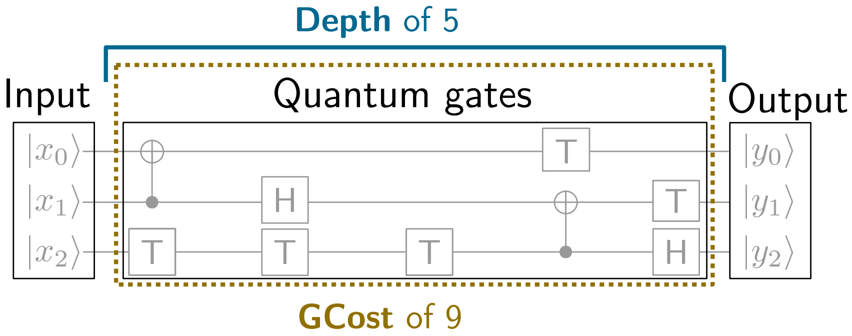
Resources: The quantum circuit model

- **GCost**: Number of universal quantum gates ▷ *lower bound on computation*



Resources: The quantum circuit model

- **GCost**: Number of universal quantum gates ▷ *lower bound on computation*
- **Depth**: Circuit depth ▷ *lower bound on time*



Infeasible amount

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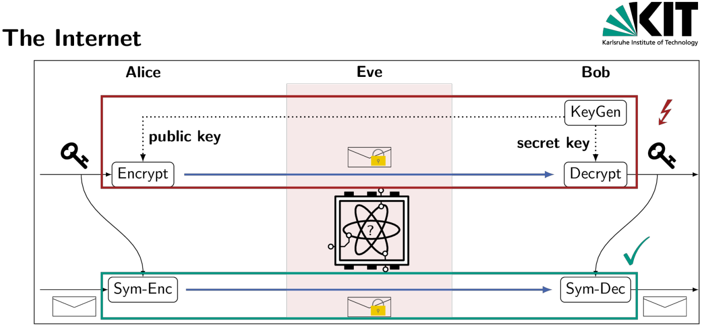
a) Advanced Encryption Standard (AES) believed to be *quantum-secure*[1]



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Infeasible amount

Karlsruhe Institute of Technology

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Kyber *quantum-secure*[2],
if **GCost**("attacking Kyber") $\geq$ **GCost**("attacking AES")

AES-128	Kyber-512
AES-192	Kyber-768
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[2,3] National Institute for Standards and Technology 2017, Post-Quantum Cryptography Call for Proposals

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b) NIST's hypothetical $\textsc{MaxDepth} \in \{2^{40}, 2^{64}, 2^{96}\}$ for **Depth**

"number of gates [...] quantum computing [...] expected to serially perform [...]"[3]

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When is a cryptosystem quantum-secure?

Kyber ↓   Shortest Vector Problem (SVP) ↓   quantum enumeration ✓

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$$\textbf{GCost}(\text{“quantum enumeration”}) \geq \textbf{GCost}(\text{“attacking AES”})$$
$$\textbf{Depth}(\text{“quantum enumeration”}) \leq \textsc{MaxDepth}$$ ✓

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Contribution: Lower bound on quantum enumeration

Enumeration as quantum walk: $\#\mathrm{QW} \times \underbrace{O\left(\sqrt{\#\mathcal{T} \cdot n}\right) \times \mathcal{W}}_{\text{quantum walk}}$

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- **GCost**$(\textsc{QEnum}) \quad = \#\mathrm{QW} \cdot O\left(\sqrt{\#\mathcal{T}} \cdot n\right) \cdot$ **GCost**$(\mathcal{W})$

Contribution: Lower bound on quantum enumeration

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Asymptotic lower bounds ✓
Heuristics, experiments
Constant/ polynomial factors
...

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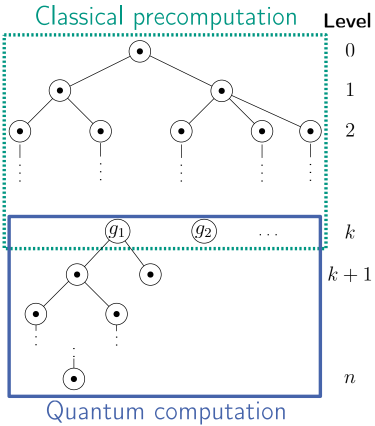
Asymptotic lower bounds
Heuristics, experiments
Constant/ polynomial factors
...
✓

- Restriction: **Depth** $\leq \textsc{MaxDepth} \in \{2^{40}, 2^{64}, 2^{96}\}$: Adapt algorithm. ✓

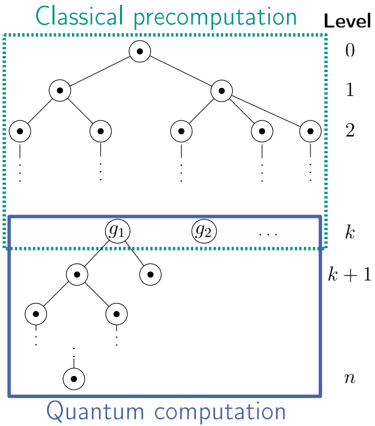
A new quantum-classical algorithm (simplified)

- Classical precomputation: up to level k



A new quantum-classical algorithm (simplified)

- Classical precomputation: up to level k
- QENUM for every node g_i on level k

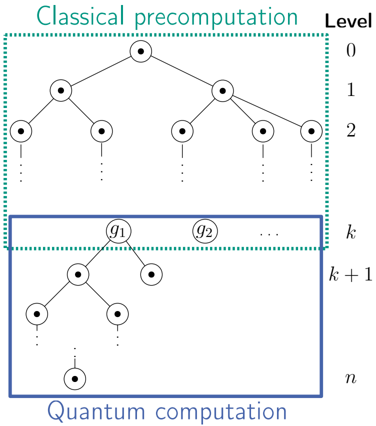


A new quantum-classical algorithm (simplified)

- Classical precomputation: up to level k
- $\textsc{QEnum}$ for every node g_i on level k
- Choose level k such that

$$\textbf{Depth}(\textsc{QEnum}) \leq \textsc{MaxDepth}$$

… and also reducing overall **GCost**.



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- $\textsc{QEnum}$ for every node g_i on level k
- Choose level k such that

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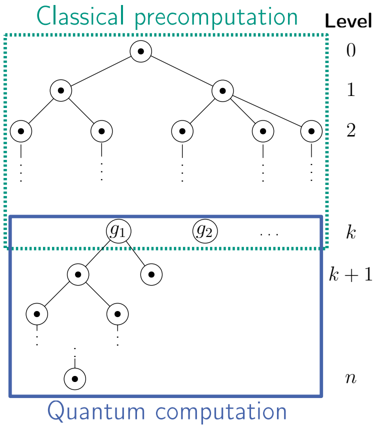
… and also reducing overall **GCost**.

$$\textbf{Total Cost} = \text{Classical precomputation} + \sum_{\text{for each } g_i \text{ on level k}} \textbf{GCost}(\textsc{QEnum})$$



Simplified results

$$\log(\text{Lower bound}(\textbf{Total Cost})) \overset{?}{\geq} \log(\textbf{GCost}(\text{"attacking AES"}))$$

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Simplified results

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MaxDepth	Kyber-512 AES-128	Kyber-768 AES-192	Kyber-1024 AES-256
2^{40}	$94 < 117$		
2^{64}	$75 < 93$		
2^{96}	$75 < 83$		

quantum enumeration

cheaper than **GCost**(“attacking AES”)

This does not mean that Kyber-512 is insecure!

Simplified results

$$\log(\text{Lower bound}(\textbf{Total Cost})) \overset{?}{\geq} \log(\textbf{GCost}(\text{“attacking AES”}))$$

MaxDepth	Kyber-512 AES-128	Kyber-768 AES-192	Kyber-1024 AES-256
2^{40}	$94 < 117$	$197 > 181$	$312 > 245$
2^{64}	$75 < 93$	$173 > 157$	$288 > 221$
2^{96}	$75 < 83$	$143 > 125$	$232 > 189$

quantum enumeration

Kyber-512: cheaper than **GCost**(“attacking AES”)

Kyber-768 and Kyber-1024: more expensive than **GCost**(“attacking AES”)

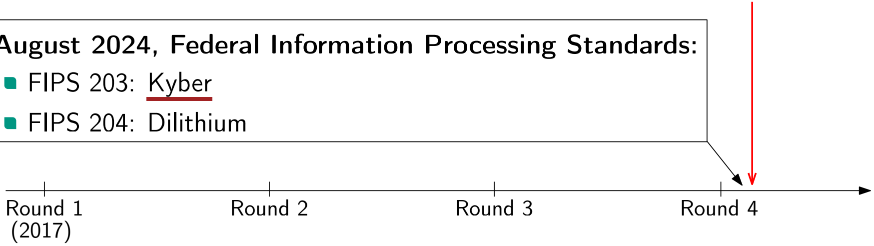
This does not mean that Kyber-512 is insecure!

Impact on post-quantum standards

Lattice Enumeration in Limited Depth
Advances in Cryptology – CRYPTO 2024

August 2024, Federal Information Processing Standards:

- FIPS 203: Kyber
- FIPS 204: Dilithium



Impact on post-quantum standards

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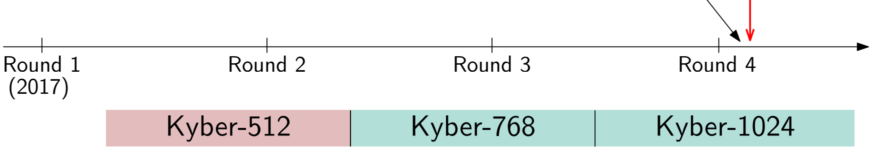

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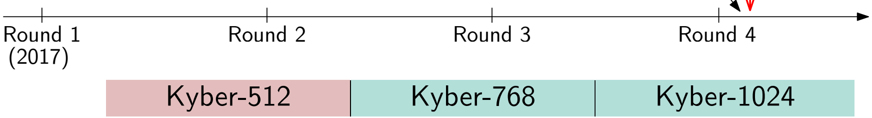
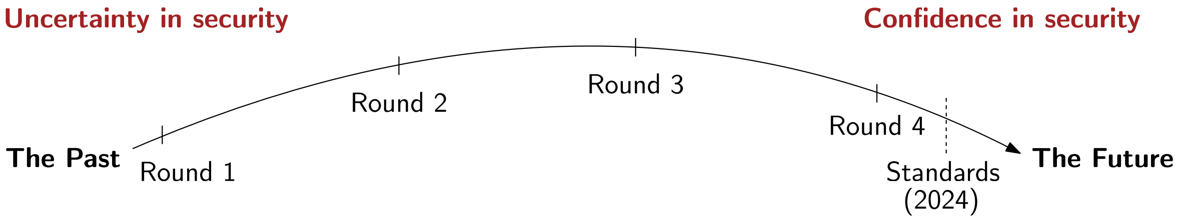
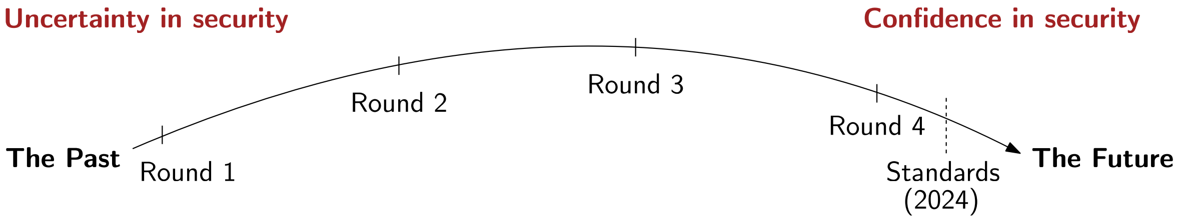


Lattice-based cryptography is **secure** for large parameters.
("The Internet" and civil aviation are saved)

A bridge to the future

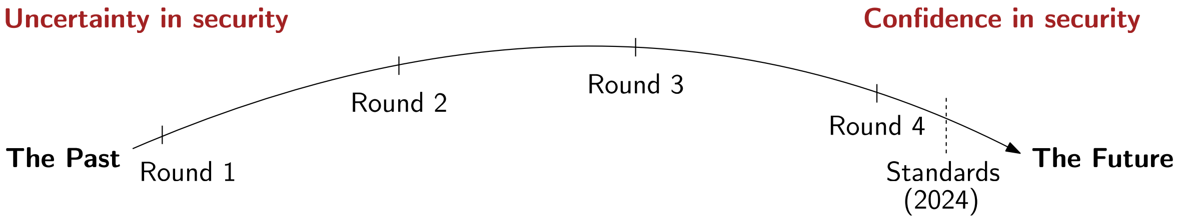


A bridge to the future



Research advances understanding and provides confidence in quantum-secure cryptography.

A bridge to the future



Research advances understanding and provides confidence in quantum-secure cryptography.

Uncertainty remains a challenge for new technologies, applications and protocols.

On to many more bridges

Exploiting Decryption Failures in Mersenne Number Cryptosystems
PKC at AsiaCCS 2020

Lattice Enumeration in Limited Depth
Advances in Cryptology – CRYPTO 2024

Forging SPHINCS$^+$-Haraka Signatures on a Fault-Tolerant Quantum Computer
Progress in Cryptology – Latincrypt 2021

Round 1 (2017) | Round 2 | Round 3 | Round 4

Making an Asymmetric PAKE Quantum-Annoying by Hiding Group Elements
ESORICS 2023

Post-Quantum Ready Key Agreement for Aviation
Communications in Cryptology 2024

My research advances cryptography to protect our digital future.

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